

Un peu de mathématiques en physique, et autres géométries inattendues

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11 Avril 2023,
Fianarantsoa

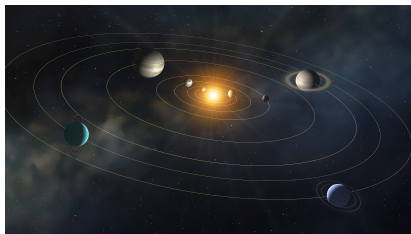
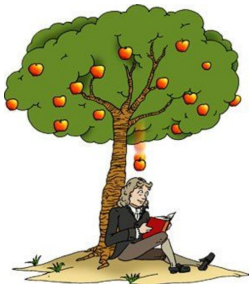
L'infiniment grand et l'infiniment petit

La déraisonnable efficacité des mathématiques

1. Vidéo cosmic eye

Mécanique : l'horlogerie du monde

Mécanique Newtonienne



$$\vec{F} = m\vec{a}$$

$$\vec{F}_g = G \frac{m_1 m_2}{d^2}$$

Mécanique Newtonienne : Prédictions – Mécanique orbitale

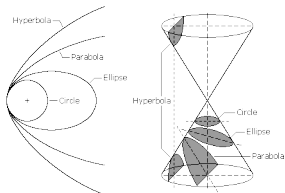
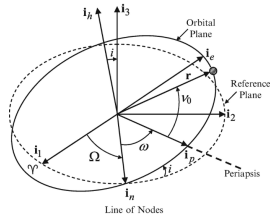
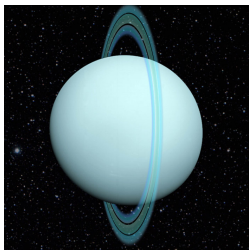


Figure 4.1

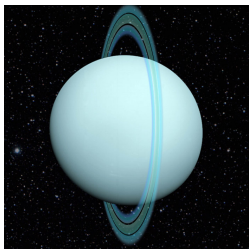


Découverte de Neptune

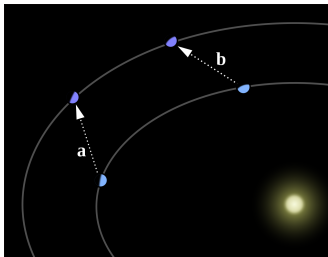


Uranus

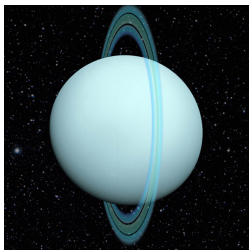
Découverte de Neptune



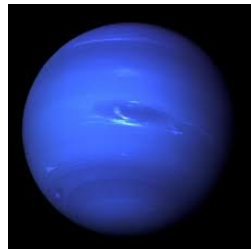
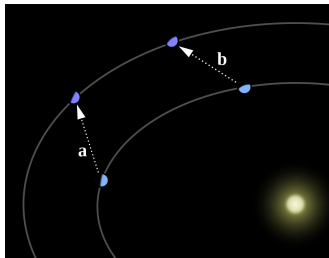
Uranus



Découverte de Neptune

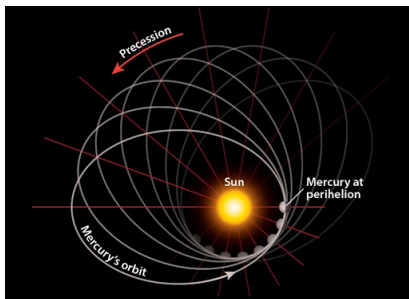


Uranus

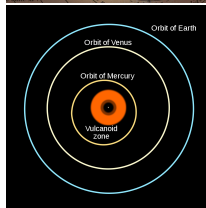
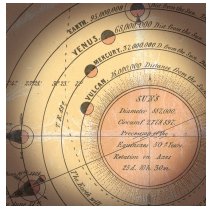
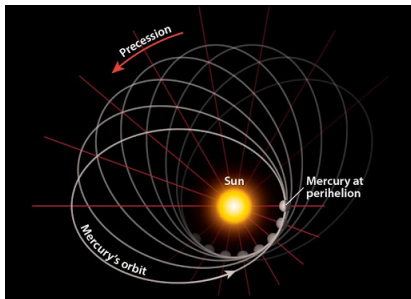


Neptune

Précession du périhélie de Mercure



Précession du périhélie de Mercure

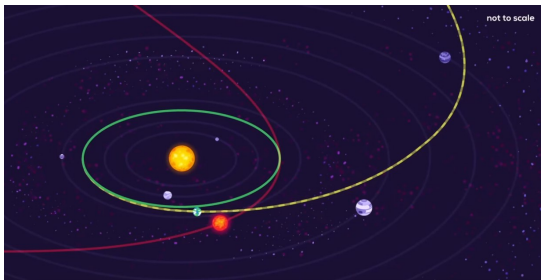


Vulcan ?

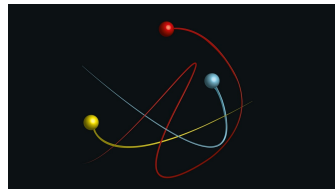
Des tas d'applications...



... mais encore beaucoup d'inconnues



Stabilité du système solaire ?

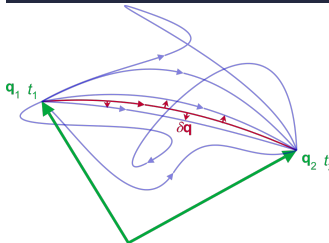
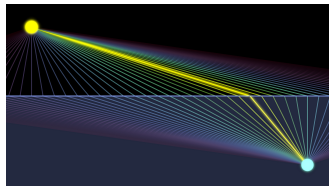


Problème à 3 corps

Mécanique Lagrangienne – Principe de moindre action

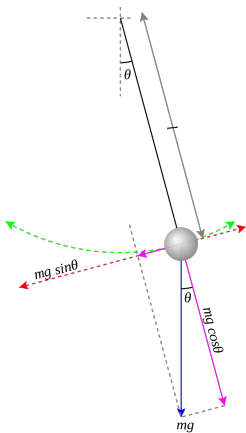


$$\mathcal{L}(q_i, \dot{q}_i, t) = E_c - E_p$$

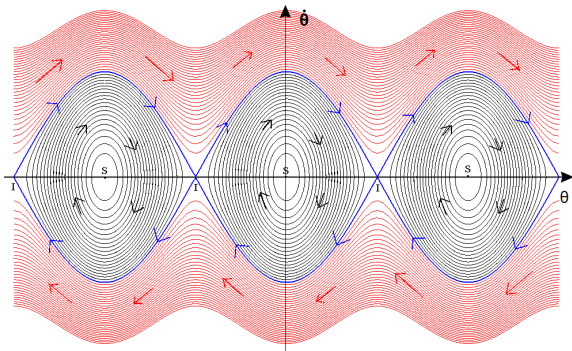


« Lorsqu'il arrive quelque changement dans la nature, la quantité d'action, nécessaire pour ce changement, est la plus petite qui soit possible. »

Mécanique Hamiltonienne – Espace des phases

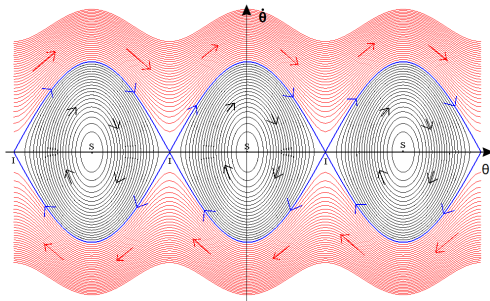
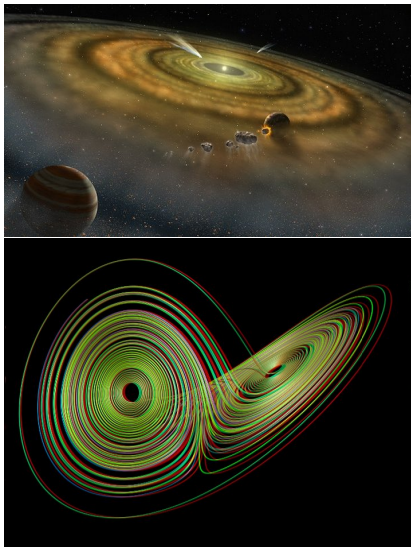


Pendule

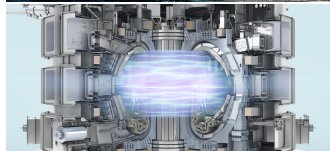
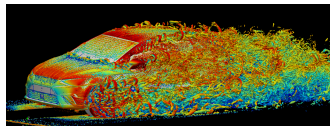


$$p_i := \frac{\partial \mathcal{L}}{\partial \dot{q}_i}, \quad \mathcal{H}(q_i, p_i, t) = \sum \dot{q}_k p_k - \mathcal{L}(q_i, \dot{q}_i, t)$$

Théorie du chaos, des perturbations, ...



Une question contemporaine : mécanique des fluides



Navier-Stokes

$$\nabla \cdot u = 0$$

$$\rho \frac{du}{dt} = -\nabla p + \mu \nabla^2 u + F$$

Equations

L'univers et ses symétries

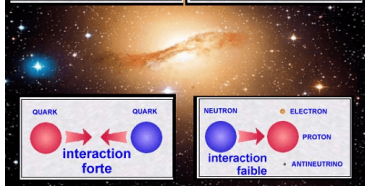
Une introduction physique à la théorie des groupes

La physique de 2023...

La physique de 2023... C'est compliqué

$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\mu g_\nu^\rho \partial_\nu g_\rho^\mu - g_\mu f^{abc} \partial_\nu g_\rho^\mu g_\nu^\rho g_\sigma^\mu g_\sigma^\rho - \frac{1}{4}g_\mu^2 f^{abc} f^{ade} g_\nu^\mu g_\nu^\rho g_\sigma^\mu g_\sigma^\rho - \partial_\mu W_\nu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\mu Z_\nu^0 \partial_\nu Z_\mu^0 - \frac{1}{2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\nu A_\mu - igc_w(\partial_\mu Z_\nu^0(W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0(W_\mu^+ \partial_\mu W_\nu^- - W_\nu^- \partial_\mu W_\mu^+) + Z_\nu^0(W_\mu^+ \partial_\mu W_\nu^- - W_\nu^- \partial_\mu W_\mu^+)) - \\
 & ig_s(\partial_\mu A_\nu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu(W_\mu^+ \partial_\mu W_\nu^- - W_\nu^- \partial_\mu W_\mu^+) + A_\nu(W_\nu^+ \partial_\mu W_\mu^- - \\
 & W_\mu^- \partial_\nu W_\nu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\nu^+ W_\mu^- + g^2 s_w^2(Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^- - \\
 & Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2(A_\mu W_\nu^+ A_\nu W_\mu^- - A_\mu A_\nu W_\nu^+ W_\mu^-) + g^2 s_w c_w(A_\mu Z_\nu^0(W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\nu Z_\mu^0 W_\nu^+ W_\mu^-) - \frac{1}{2}\partial_\mu H \partial_\nu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\nu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\nu \phi^0 - \\
 & \beta_h \left(\frac{2M_h^2}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M_h^2}{g^2} \alpha_h - \\
 & \frac{g\alpha_h M}{2} (H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-) - \\
 & \frac{1}{2}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 & gM W_\mu^+ W_\nu^- H - \frac{1}{2}g \frac{g^2}{c_w} Z_\mu^0 Z_\nu^0 H - \\
 & \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\nu \phi^- - \phi^- \partial_\nu \phi^0) - W_\nu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
 & \frac{1}{2}g (W_\mu^+ (H\partial_\nu \phi^- - \phi^- \partial_\nu H) + W_\nu^- (H\partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H\partial_\nu \phi^0 - \phi^0 \partial_\nu H) + \\
 & M (\frac{1}{c_w} Z_\mu^0 \partial_\nu \phi^0 + W_\mu^+ \partial_\nu \phi^- + W_\nu^- \partial_\mu \phi^+)) - ig \frac{g^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\nu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\
 & W_\nu^- \phi^+) - ig \frac{1-2s_w^2}{c_w} Z_\mu^0 (\phi^+ \partial_\nu \phi^- - \phi^- \partial_\nu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\nu \phi^- - \phi^- \partial_\nu \phi^+) - \\
 & \frac{1}{2}g^2 W_\mu^+ W_\nu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{2}g^2 \frac{1}{c_w} Z_\mu^0 Z_\nu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 & \frac{1}{2}g^2 \frac{g^2}{c_w} Z_\mu^0 (W_\nu^+ \phi^- + W_\nu^- \phi^+) - \frac{1}{2}ig \frac{g^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\nu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\nu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\nu^- \phi^+) - g^2 \epsilon \epsilon (2c_w^2 - 1) Z_\mu^0 A_\nu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\nu \phi^+ \phi^- + \frac{1}{2}ig_\mu \lambda_\mu^2 (\tilde{g}_\mu^+ \tilde{g}_\mu^+ \tilde{g}_\mu^+) g_\mu^2 - e^3 (\gamma\theta + m_\Delta^2) e^3 - e^3 (\gamma\theta + m_\Delta^2) \nu^3 - \mu_\Delta^2 (\gamma\theta + \\
 & m_\Delta^2) u_\Delta^3 - d_\Delta^2 (\gamma\theta + m_\Delta^2) d_\Delta^3 + ig s_w A_\mu (- (e^3 \gamma^\mu e^3) + \frac{2}{3}(\tilde{u}_\Delta^3 \gamma^\mu u_\Delta^3) - \frac{1}{3}(\tilde{d}_\Delta^3 \gamma^\mu d_\Delta^3)) + \\
 & \frac{ig}{2M} Z_\mu^0 \{ (\tilde{\nu}^3 \gamma^\mu (1 + \gamma^5) \nu^3) + (e^3 \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^3) + (d_\Delta^3 \gamma^\mu (\frac{2}{3}s_w^2 - 1 - \gamma^5) d_\Delta^3) + \\
 & (\tilde{u}_\Delta^3 \gamma^\mu (1 - \frac{2}{3}s_w^2 + \gamma^5) u_\Delta^3) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ \{ (\tilde{\nu}^3 \gamma^\mu (1 + \gamma^5) U^{lep}_{\lambda\mu} e^3) + (\tilde{u}_\Delta^3 \gamma^\mu (1 + \gamma^5) C_{\lambda\mu} d_\Delta^3) \} + \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- \{ (e^3 U^{lep}_{\lambda\mu} \gamma^\mu (1 + \gamma^5) \nu^3) + (\tilde{d}_\Delta^3 C_{\lambda\mu} \gamma^\mu (1 + \gamma^5) u_\Delta^3) \} + \\
 & \frac{ig}{2M} \phi^+ (-m_\Delta^2 (\tilde{\nu}^3 U^{lep}_{\lambda\mu} (1 - \gamma^5) e^3) + m_\Delta^2 (\tilde{\nu}^3 U^{lep}_{\lambda\mu} \lambda_\mu (1 + \gamma^5) e^3) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_\Delta^2 (e^3 U^{lep}_{\lambda\mu} (1 + \gamma^5) \nu^3) - m_\Delta^2 (e^3 U^{lep}_{\lambda\mu} (1 - \gamma^5) \nu^3) - \frac{g}{2} \frac{m_\Delta^2}{M} H (\tilde{\nu}^3 \nu^3) - \\
 & \frac{g}{2} \frac{m_\Delta^2}{M} H (e^3 e^3) + \frac{ig}{2} \frac{m_\Delta^2}{M} \phi^0 (\tilde{\nu}^3 \nu^3 \nu^3) - \frac{ig}{2} \frac{m_\Delta^2}{M} \phi^0 (e^3 \nu^3 e^3) - \frac{1}{2} \nu_\lambda M_\mu^R (1 - \gamma_5) \partial_\mu - \\
 & \frac{1}{2} \nu_\lambda M_\mu^R (1 - \gamma_5) \partial_\mu + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_\Delta^2 (\tilde{u}_\Delta^3 C_{\lambda\mu} (1 - \gamma^5) d_\Delta^3) + m_\Delta^2 (\tilde{u}_\Delta^3 C_{\lambda\mu} (1 + \gamma^5) d_\Delta^3) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_\Delta^2 (\tilde{d}_\Delta^3 C_{\lambda\mu}^+ (1 + \gamma^5) u_\Delta^3) - m_\Delta^2 (\tilde{d}_\Delta^3 C_{\lambda\mu}^- (1 - \gamma^5) u_\Delta^3) - \frac{g}{2} \frac{m_\Delta^2}{M} H (\tilde{u}_\Delta^3 u_\Delta^3) - \\
 & \frac{g}{2} \frac{m_\Delta^2}{M} H (\tilde{d}_\Delta^3 d_\Delta^3) + \frac{ig}{2} \frac{m_\Delta^2}{M} \phi^0 (\tilde{u}_\Delta^3 \gamma^5 u_\Delta^3) - \frac{ig}{2} \frac{m_\Delta^2}{M} \phi^0 (\tilde{d}_\Delta^3 \gamma^5 d_\Delta^3) + \tilde{G}^\alpha \tilde{\theta}^\alpha G^\alpha + g_\mu f^{abc} \partial_\nu \tilde{G}^\alpha \tilde{G}^\alpha g_\mu^2 - \\
 & \tilde{X}^\alpha (\tilde{\theta}^2 - M^2) X^\alpha + \tilde{X}^- (\tilde{\theta}^2 - M^2) X^- + X^0 (\tilde{\theta}^2 - \frac{M^2}{c_w^2}) X^0 + Y \tilde{\theta}^2 Y + igc_w W_\mu^+ (\partial_\mu \tilde{X}^0 X^0 - \\
 & \partial_\mu \tilde{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \tilde{Y} X^- - \partial_\mu \tilde{X}^+ Y) + igc_w W_\mu^- (\partial_\mu \tilde{X}^- X^0 - \\
 & \partial_\mu \tilde{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \tilde{X}^- Y - \partial_\mu \tilde{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \tilde{X}^+ X^+ - \\
 & \partial_\mu \tilde{X}^- X^-) + ig s_w A_\mu (\partial_\mu \tilde{X}^+ X^+ - \\
 & \partial_\mu \tilde{X}^- X^-) - \frac{1}{2}gM (\tilde{X}^+ X^+ H + \tilde{X}^- X^- H + \frac{1}{2} \tilde{X}^0 X^0 H) + \frac{1-2s_w^2}{2c_w} igM (\tilde{X}^+ X^0 \phi^+ - \tilde{X}^- X^0 \phi^-) + \\
 & \frac{1}{2c_w} igM (\tilde{X}^0 X^- \phi^+ - \tilde{X}^0 X^+ \phi^-) + igM s_w (\tilde{X}^0 X^- \phi^+ - \tilde{X}^0 X^+ \phi^-) + \\
 & \frac{1}{2}igM (\tilde{X}^+ X^0 \phi^+ - \tilde{X}^- X^0 \phi^-) .
 \end{aligned}$$

La physique de 2023... C'est compliqué ?



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + h.c. + \bar{\psi}_i \gamma_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

THE STANDARD MODEL OF PARTICLE PHYSICS

THE STANDARD MODEL

QUARKS: UP, CHARM, TOP, DOWN, STRANGE, BOTTOM

LEPTONS: ELECTRON, MUON, TAU

GAUGE BOSONS: PHOTON, Z BOSON, W BOSON, GLUON

ALL THE FUNDAMENTAL PARTICLES

CONSERVATION LAWS: ENERGY, CHARGE, BARYON NUMBER, LEPTON NUMBERS, STRANGENESS

WEAK FORCE

ELECTROMAGNETIC FORCE

STRONG FORCE

FORCE INTERACTIONS: INTERACT WITH THE ELECTROMAGNETIC FORCE, INTERACT WITH THE STRONG FORCE, INTERACT WITH THE WEAK FORCE, INTERACT WITH THE HIGGS FIELD

STANDARD MODEL INTERACTIONS: 1. ALL QUARKS INTERACT WITH ALL GAUGE BOSONS. 2. ALL LEPTONS INTERACT WITH THE PHOTON AND Z BOSON. 3. ALL QUARKS INTERACT WITH ALL GLUONS. 4. ALL QUARKS AND LEPTONS INTERACT WITH THE W AND Z BOSONS. 5. ALL QUARKS AND LEPTONS INTERACT WITH THE HIGGS BOSON.

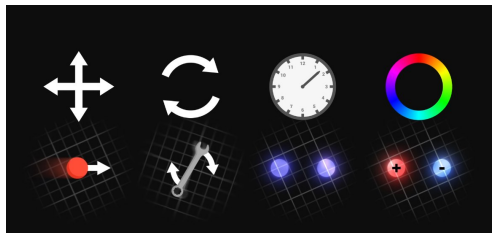
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L'idée géniale d'Emmy Noether

$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = H|\Psi\rangle$
 $U \cdot \nabla U = 0$
 $\vec{F}_{\text{ext}} = m\vec{a}$
 $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$

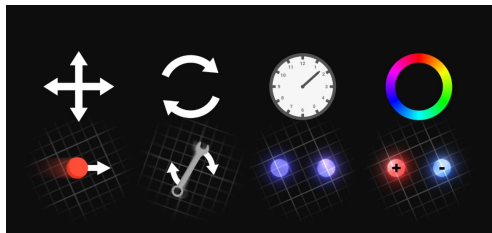
$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = H|\Psi\rangle$
 $U \cdot \nabla U = 0$
 $\sum \vec{F}_{\text{ext}} = m\vec{a}$
 $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$

L'idée géniale d'Emmy Noether



« À toute symétrie de l'univers correspond une grandeur physique conservée »

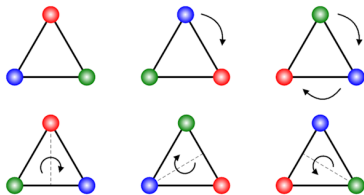
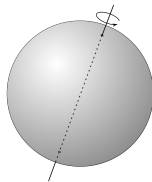
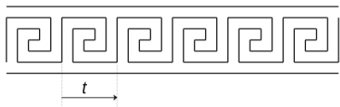
L'idée géniale d'Emmy Noether



« À toute symétrie de l'univers correspond une grandeur physique conservée »

Pour comprendre les lois de l'univers, il suffit de trouver ses symétries.

Mais qu'est-ce qu'une symétrie ?



Une symétrie est une transformation qui laisse le système inchangé.
 Pour un système, on peut étudier toutes les manières dont il est symétrique, et la manière dont ces symétries se composent : c'est son groupe de symétrie.

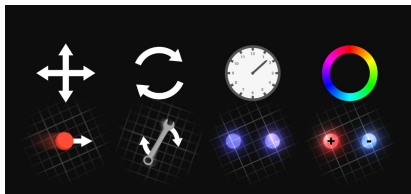
Symétrie de l'univers

Quand les lois de l'univers sont invariantes par une transformation.



Symétrie de l'univers

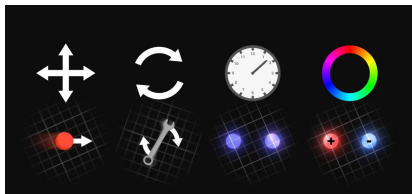
Quand les lois de l'univers sont invariantes par une transformation.



- Et la vitesse de la lumière...

Symétrie de l'univers

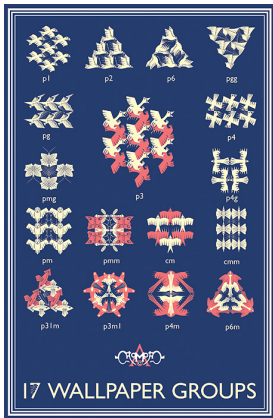
Quand les lois de l'univers sont invariantes par une transformation.



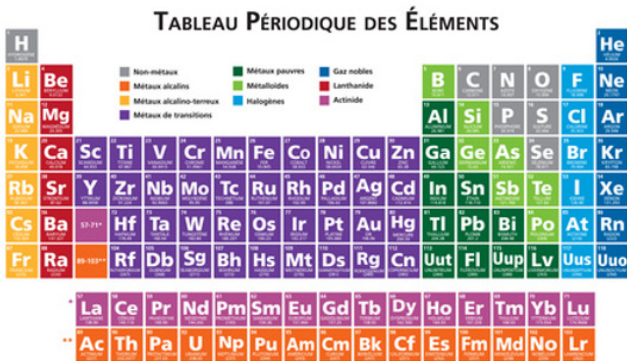
- Et la vitesse de la lumière...
- D'autres symétries microscopiques.
 $U(1)$ pour l'électromagnétisme, $SO(2)$ pour l'interaction faible, $SO(3)$ pour les symétries de couleur de la chromodynamique des quarks, ...

La symétrie chez les mathématiciens (1)

« Les 17 groupes de papier peint »



La symétrie chez les mathématiciens (2)



La symétrie chez les mathématiciens (2)

The Periodic Table Of Finite Simple Groups

B,C,F ₄		Dynkin Diagrams of Simple Lie Algebras														C ₂						
1																2						
A ₁ (4), A ₃ (3)		A ₅ (2)		A ₇ (2)		A ₉ (2)		A ₁₁ (2)		A ₁₃ (2)		A ₁₅ (2)		A ₁₇ (2)		A ₁₉ (2)		A ₂₁ (2)		A ₂₃ (2)		C ₃
60		360		2520		10080		362880		1344000		47900160		1724972800		63551344000		2311016064000		8426961600000		5
A ₁ (6), A ₂ (2), A ₃ (2)		A ₄ (3)		A ₅ (3)		A ₆ (3)		A ₇ (3)		A ₈ (3)		A ₉ (3)		A ₁₀ (3)		A ₁₁ (3)		A ₁₂ (3)		A ₁₃ (3)		C ₅
360		360		360		360		360		360		360		360		360		360		360		5
A ₇		A ₁₁ (2)		E ₇ (2)		E ₈ (2)		G ₂ (3)		³ D ₄ (2 ²)		² E ₆ (2 ²)		2B ₂ (2 ³)		2F ₄ (2 ¹)		2G ₂ (3 ³)		B ₃ (2)		C ₇
2520		360		420		420		10080		10080		10080		10080		10080		10080		10080		7
A ₉		A ₁₃ (2)		E ₆ (3)		E ₇ (3)		E ₈ (3)		G ₂ (4)		³ D ₄ (3 ²)		2E ₆ (3 ²)		2B ₂ (2 ²)		2F ₄ (2 ²)		2G ₂ (3 ²)		C ₁₁
20160		180		180		180		180		180		180		180		180		180		180		11
A ₁₁		A ₁₇ (2)		E ₆ (4)		E ₇ (4)		E ₈ (4)		G ₂ (5)		³ D ₄ (4 ²)		2E ₆ (4 ²)		2B ₂ (2 ²)		2F ₄ (2 ²)		2G ₂ (3 ²)		C ₁₃
181440		240		240		240		240		240		240		240		240		240		240		13
A ₁₃		A ₁₉ (2)		E ₅ (5)		E ₆ (5)		E ₇ (5)		E ₈ (5)		G ₂ (6)		3D ₄ (5 ²)		2E ₆ (5 ²)		3F ₄ (2 ²⁺¹)		3G ₂ (3 ²⁺¹)		C ₁₇
151200		120		120		120		120		120		120		120		120		120		120		17
A ₁₅		A ₂₁ (2)		E ₄ (6)		E ₅ (6)		E ₆ (6)		E ₇ (6)		E ₈ (6)		G ₂ (7)		3D ₄ (6 ²)		2E ₆ (6 ²)		3F ₄ (2 ²⁺¹)		C ₁₉
120960		60		60		60		60		60		60		60		60		60		60		19

- Alternating Groups
- Classical Chevalley Groups
- Chevalley Groups
- Classical Twisted Chevalley Groups
- Steinberg Groups
- Suzuki Groups
- Lie Groups and Tits Groups*
- Sporadic Groups
- Cyclic Groups

Alternator*
Symbol

M ₁₁	M ₁₂	M ₂₂	M ₂₃	M ₂₄	J ₁	J ₂	J ₃	J ₄	HS	McL	He	Ru
7920	95040	443520	3321600	244423680	175368	668880	35322360	36288000	84513600	44352000	430016640	6478344000

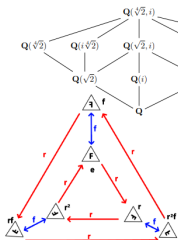
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*The Suzuki group ${}^2B_2(2)$ is not in this table.
*The Steinberg group ${}^2G_2(3)$ is not in this table.
*The Tits group ${}^2F_4(2)$ is not in this table.
*The Steinberg group ${}^2G_2(3)$ is not in this table.
*The Tits group ${}^2F_4(2)$ is not in this table.
*The Steinberg group ${}^2G_2(3)$ is not in this table.

*The Tits group ${}^2F_4(2)$ is not in this table.
*The Suzuki group ${}^2B_2(2)$ is not in this table.
*The Steinberg group ${}^2G_2(3)$ is not in this table.
*The Tits group ${}^2F_4(2)$ is not in this table.
*The Steinberg group ${}^2G_2(3)$ is not in this table.
*The Tits group ${}^2F_4(2)$ is not in this table.
*The Steinberg group ${}^2G_2(3)$ is not in this table.

S ₂	O'N	O	O	O	O	O	O	O	O	O	O	O
5040	10080	10080	10080	10080	10080	10080	10080	10080	10080	10080	10080	10080

La symétrie chez les mathématiciens (3)

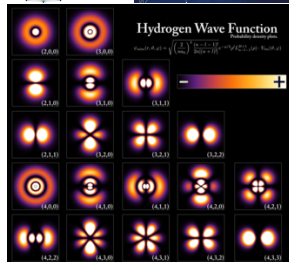
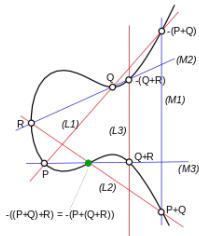
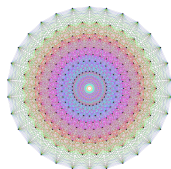
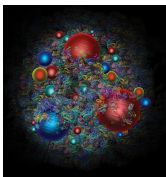
Une aventure démarrée par le jeune Galois...



$$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$$

La symétrie chez les mathématiciens (4)

Une aventure démarrée par le jeune Galois...
et toujours de nouvelles applications



Géométrie non-euclidienne

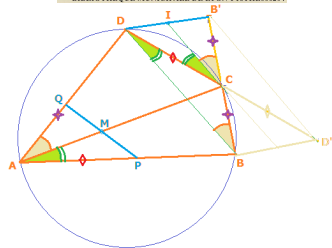
Géométrie euclidienne

23 définitions, 10 axiomes, et 5 postulats

- ① Il existe toujours une droite qui passe par deux points du plan.
- ② Tout segment peut être étendu suivant sa direction en une droite (infinie).
- ③ À partir d'un segment, il existe un cercle dont le centre est un des points du segment et dont le rayon est la longueur du segment.
- ④ Tous les angles droits sont égaux entre eux.
- ⑤ Étant donné un point et une droite ne passant pas par ce point, il existe une seule droite passant par ce point et parallèle à la première.



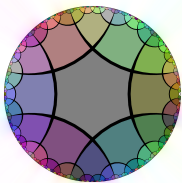
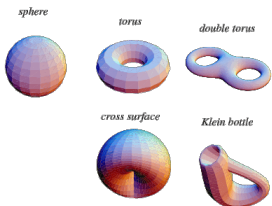
BIBLIOTHEQUE MUNICIPALE DE LYON-F16THE000204



Géométrie non-euclidienne

Changeons les règles du jeu !

- Étant donné un point et une droite ne passant pas par ce point, il existe une seule droite passant par ce point et parallèle à la première.



Flat

Triangle: sum of angles is 180° .

Parallel Lines: remain parallel.

Circle: $C = 2\pi r$.

Straightest Possible Path: is a straight line.

Spherical

Triangle: sum of angles is greater than 180° .

Parallel Lines: eventually converge.

Circle: $C < 2\pi r$.

Straightest Possible Path: is a piece of a great circle.

Hyperbolic

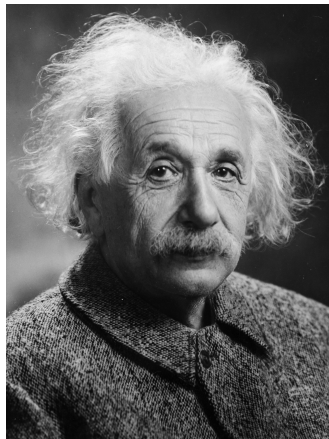
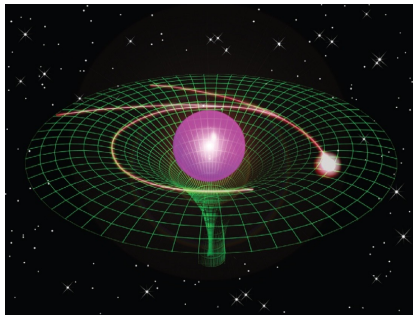
Triangle: sum of angles is less than 180° .

Parallel Lines: eventually diverge.

Circle: $C > 2\pi r$.

Straightest Possible Path: is a piece of a hyperbola.

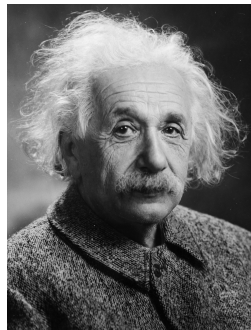
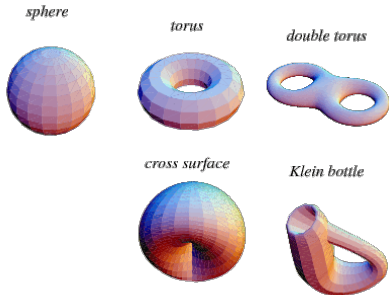
... L'univers est non-euclidien !



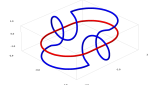
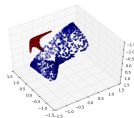
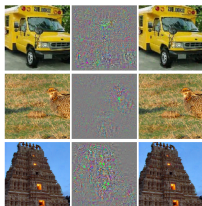
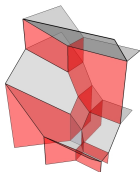
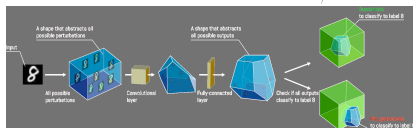
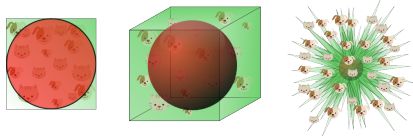
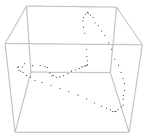
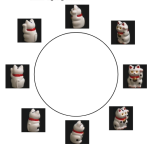
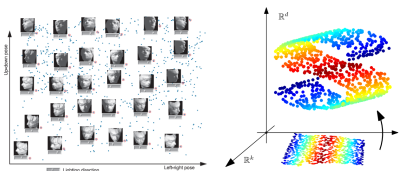
... L'univers est non-euclidien !

Autres questions :

- Quelle est sa topologie ?
- Est-il "vraiment" de dimension 3 ? Ou 4 ?
Ou 11 ?



Ouverture(s) : géométrie et topologie en informatique



Ouverture(s) : physique statistique

Faire le lien entre infiniment petit et infiniment grand

Conclusion

Conclusion

Merci !