Research Introduction

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PLIS Group, Roskilde University

CLIP Lab @ IMDEA Software Institute, Madrid

IMDEA Software Institute

Madrid Institute for Advanced Studies
Focus: *Safe*, *Reliable* and *Efficient* Software

Program Analysis and Verification,
Cryptography and Cybersecurity,
Languages and Compilers, ...

CLIP Lab

The **C**omputational logic, **L**anguages,
Implementation, and **P**arallelism **Lab**oratory

Manuel Hermenegildo

Pedro Lopez-Garcia

Jose F. Morales

Manuel Carro

Daniel Jurjo

Daniela Ferreiro

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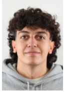


Marco Ciccale

Paula Corral

Kirelys Lugo

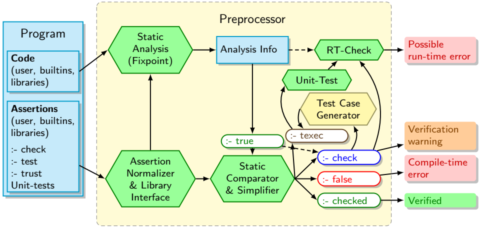
An Extensible Logic Programming Language,
designed to make full use of advanced
Analysis, **Verification** and **Optimisation**

CLIP Lab @ IMDEA Software Institute, Madrid

```
:- module(_, [nrev/2], [assertions,fsyntax,nativeprops]).

:- entry nrev/2 : {list, ground} * var.

:- pred nrev(A,B) : list(A) => list(B)
   + ( not_fails, is_det, steps_o( length(A)) ).

nrev( [] )    := [].
nrev( [H|L] ) := ~conc( ~nrev(L),[H] ).


:- pred conc(A,B,C) + ( terminates, is_det, steps_o(length(A)) ).

conc( [],     L ) := L.
conc( [H|L], K ) := [ H | ~conc(L,K) ].
```

Preprocessor Option Browser

Use Saved Menu Configuration (menu_last_config)	:	none
Menu Level (menu_level)	:	naive
Action (inter_all)	:	check_assertions
Analysis Domain (assert_ctcheck)	:	manual
Modules to Check (ct_modular)	:	curr_mod
Customize Analysis Flags (check_config_ana)	:	on
Analyze Non-Failure (ana_nf)	:	nf
Analyze Numeric (ana_num)	:	none
Analyze Cost (ana_cost)	:	none
Analyze Determinism (ana_det)	:	det
Analysis entry (entry_point)	:	entry
Incremental (incremental)	:	off
Intermodular (intermod)	:	off
Report Non-Verified Assrts (ass_not_stat_eval)	:	warning
Generate Certificate (gen_certificate)	:	off
Generate CT Checking Intervals (ctchecks_intervals)	:	on
Generate Output (menu_output)	:	on
| Output Language (output_lang)	:	source
| Include Program Point (pp_info)	:	off
| Multi-variant Analysis Results (vers)	:	off
| Collapse Versions (collapse_ai_vers)	:	on




```
#pragma check fir(xn, coeffs, state, ELEMENTS) :
          (1 <= ELEMENTS && energy <= 416.0)
#pragma true fir(xn, coeffs, state, ELEMENTS) :
          ( energy >= 3.35*ELEMENTS + 13.96 &&
            energy <= 3.35*ELEMENTS + 14.4 )
#pragma checked fir(xn, coeffs, state, ELEMENTS) :
          (1 <= ELEMENTS && ELEMENTS <= 120 && energy <= 416.1)
#pragma false fir(xn, coeffs, state, ELEMENTS) :
          (121 <= ELEMENTS && energy <= 416.1)

int fir(int xn, int coeffs[], int state[], int ELEMENTS)
{ unsigned int ynl; int ynh;
  ynl = (1<<23); ynh = 0;
  for(int j=ELEMENTS-1; j!=0; j--) {
      state[j] = state[j-1];
      {ynh, ynl} = macs(coeffs[j], state[j], ynh, ynl); }
  state[0] = xn;
  {ynh, ynl} = macs(coeffs[0], xn, ynh, ynl);
  if (sext(ynh,24) == ynh) {
      ynh = (ynh << 8) | (((unsigned) ynl) >> 24);}
  else if (ynh < 0) { ynh = 0x80000000; }
  else { ynh = 0x7fffffff; }
  return ynh; }
```

PhD Research Topic

Abstract Interpretation-based **Static Analysis of Cost Properties** (resource consumption) of software, with a particular focus on **Energy Consumption**

Cost Analysis: Bounds on Resource Consumption

Program $\mathcal{P}$

```
def foo(lst, x) =
    <body>
```

\+

Cost Model

Cost Analysis

Cost Bounds

$$\mathbf{f}_{\mathbf{lb}}(\vec{n}) \leq \mathbf{Cost}_{\texttt{foo}} \leq \mathbf{f}_{\mathbf{ub}}(\vec{n})$$

Size Abstraction

$n_1 = \texttt{len(lst)}$, $n_2 = \texttt{val(x)}$, ...

WCET,
Side-Channel

Parallelisation,
Scheduling

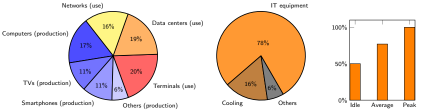
Transparency, Optimisation

PhD Research Topic

Abstract Interpretation-based **Static Analysis of Cost Properties** (resource consumption) of software, with a particular focus on **Energy Consumption**

Key personal motivation, in terms of societal impact: **carbon footprint of IT**.

Share of global emissions:
2.5% $\rightsquigarrow$ **5%** in the last ten years.



Energy Usage in IT. By subfield, and case studies on a data center.

Energy consumption *of programs* is relevant and understudied by carbon audit experts.

Previous collaboration: **ENTRA project**
ENergy **TRA**nsparency

Also in contact with associations on evolution of carbon norms and regulations.

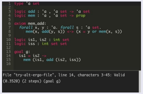
Personal Research Interests and Background

Background

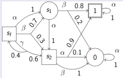
- **2016–2018**, Preparatory Classes, Mathematics / Physics
  (Louis-le-Grand, Paris, France)
- **2018–2022**, École Polytechnique, MSc
  Multidisciplinary school of Engineering and Public Administration
  Major: Theoretical Computer Science and Mathematics
- **2021–2022**, MPRI, Formal Methods and Programming Languages, MSc
  Joint degree ENS/Polytechnique/Université de Paris

Selected Past Projects (pre-PhD)

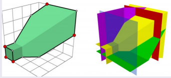
Computer Science



SMT-Solver Alt-Ergo

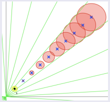


Categorical Approaches
on Model Checking



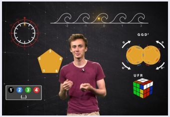
Abstract Interpretation of Neural Networks
using Tropical Geometry [SAS21]

Mathematics Outreach



Undergraduate book on S-unit equations
Geometric and Algebraic Number Theory

High-school
Research Tournaments



High-school MOOCs

Personal Research Interests and Background

Current Research Interests

Static Analysis of Programs and Systems

- Automated discovery of truths, beyond verification: *automated understanding and improvement*
- **Abstract Interpretation**
- **“Bounding the behaviour of systems”,** beyond software: hardware/networks, or even cyberphysical systems, biochemical reaction networks

Applied Semantics

- Denotational Approaches
- Abstraction Methods
- **Order Theory**

Geometric Viewpoints

(Generalised) Recurrence Equations

i.e. functional discrete equations, $f : \mathbb{Z}^k \to \mathbb{R}$.

- Accurate, executable abstractions of systems
- Non-linear invariant inference
- Non-standard equations (CAS vs programs)

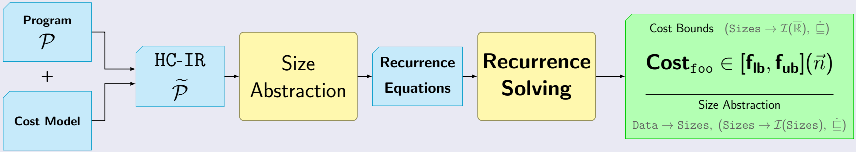
Dynamic/Static Cooperations

- Machine Learning + Formal Methods
- Observation + Reasoning
- (Optimisation/Search) + (Logic/Order)

PhD Research Topic

Abstract Interpretation-based **Static Analysis of Cost Properties** (resource consumption) of software, with a particular focus on **Energy Consumption**

Our cost analysis pipeline



Challenges

- Fine-grained (energy) cost models, hardware behaviours, etc.
  `[WIP]`. C.f. [ENTRA], WCET literature.
- Complex control flows, general programs
  [RustenholzMSc22, TPLP24, SAS24]. `[WIP]`.
- State of the art in recurrence solvers
  [op.cit.].
- Improve size abstractions
  [RidouxMSc24]. `[WIP]`.

Generalised Recurrence Equations

i.e. functional discrete equations, $f : \mathbb{Z}^k \to \mathbb{R}$. (Undecidable: must aim for bounds.)

CAS vs Generalised Equations

$$f(n) = a(n) \cdot f(\mathbf{n-1}) + b(n)$$

complex factors, simple recursive calls

(classical computer algebra)

$$f(\vec{n}) = \begin{cases} \dots \\ a \cdot f(\phi(\vec{n})) + b(\vec{n}) \\ \dots \end{cases}$$

simple factors, complex recursive calls
complex control flow
(program analysis)

Conditionals, non-linear recursion, ...

$$f(\vec{n}) = \begin{cases} \dots \\ \sum_{j=1}^{k_i} \left(a_{i,j}(\vec{n}) \cdot f(\phi_{i,j}(\vec{n}))\right) + b_i(\vec{n}) & \text{if } \varphi_i(\vec{n}) \\ \dots \end{cases}$$

where $a_{i,j} \geq 0$, and $\phi_{i,j}, b_i, \varphi_i$ arbitrary.

(Unbounded) max/min

$$f(n) = \begin{cases} 0 & \text{if } n \leq 2 \\ n + \max_{1 \leq k \leq n-1} f(k) + f(n-k) & \text{if } n > 2 \end{cases}$$

Self-composition

$$f(n) = \begin{cases} f(f(n-1)) + 1 & \text{if } n > 0 \\ 0 & \text{if } n = 0 \end{cases}$$

Equation solving as pre/postfixpoint search [SAS24]

Equations $\leftrightarrow$ Operators   Solutions $\leftrightarrow$ Fixpoints   Bounds $\leftarrow$ Pre/Postfixp

Example

Search $f : \mathbb{N}^2 \to \mathbb{R}$ such that

$$f(n,c) = \boxed{\begin{cases} f(n-1,0)+n+300 & \text{if } n>0 \text{ and } c \geq 100, \\ f(n-1,c+1)+n & \text{if } n>0 \text{ and } c<100, \\ c & \text{if } n=0, \end{cases}}$$

➔

Example

Search $f : \mathbb{N}^2 \to \mathbb{R}$ such that $f = \Phi f$, i.e. $f \in \mathrm{Fixp}(\Phi)$, where

$$\Phi : (\mathbb{N}^2 \to \mathbb{R}) \to (\mathbb{N}^2 \to \mathbb{R})$$

$$f \mapsto (n,c) \mapsto \begin{cases} f(n-1,0)+n+300 & \text{if } n>0 \text{ and } c \geq 100, \\ f(n-1,c+1)+n & \text{if } n>0 \text{ and } c<100, \\ c & \text{if } n=0. \end{cases}$$

For a complete lattice, order $\mathcal{D} \to \mathbb{R}$ pointwise, and extend to $\overline{\mathbb{R}} := \mathbb{R} \cup \{\pm\infty\}$

Theorem

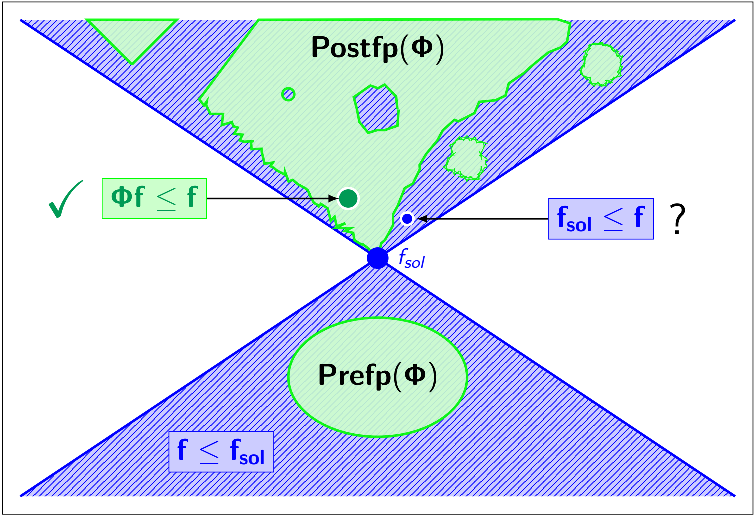
Let $\Phi : (\mathcal{D} \to \overline{\mathbb{R}}) \to (\mathcal{D} \to \overline{\mathbb{R}})$ *be a* **monotone equation**.

- *If* $f \in \mathrm{Postfp}(\Phi)$, *i.e.* $\Phi f \leq f$, *then* $\mathrm{lfp}\,\Phi \leq f$.
- *If* $f \in \mathrm{Prefp}(\Phi)$, *i.e.* $f \leq \Phi f$, *then* $f \leq \mathrm{gfp}\,\Phi$.

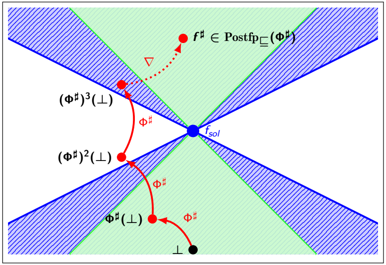
Insight: cost equations are typically monotone for this pointwise order.

Equation solving as pre/postfixpoint search [SAS24]

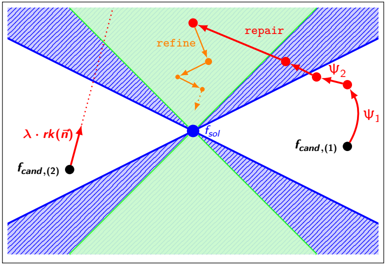
Equations $\leftrightarrow$ Operators   Solutions $\leftrightarrow$ Fixpoints   Bounds $\leftarrow$ Pre/Postfixp



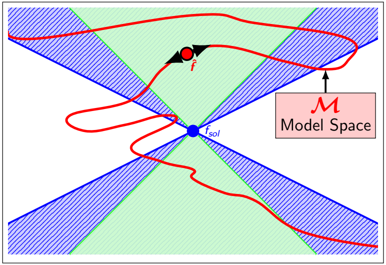
Equation solving as pre/postfixpoint search [SAS24]



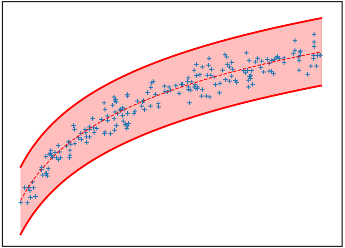
Abstract Interpretation



Geometry-based expression **Repair**



Search on subvarieties: **Templates**, ∀-elim



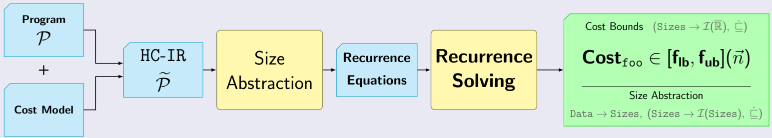
Constrained **Optimisation**,
with *provability constraints*

Interval Recurrence Equations @ Roskilde

Generalised signatures (richer domains/codomains):

- Classical: $\mathbb{Z}^k \to \mathbb{R}$
- **Intervals**: $\mathbb{Z}^k \to \mathcal{I}(\mathbb{R})$, $\quad \mathcal{I}(\mathbb{Z})^k \to \mathcal{I}(\mathbb{R})$, $\quad \mathcal{I}(\mathbb{Z})^k \to \mathcal{I}(\mathbb{Z})^r$
- With environment: $\mathsf{L}^\sharp \to \mathcal{I}(\mathbb{Z})^k \to \mathcal{I}(\mathbb{R})$

Cost analysis pipeline



- Novel size abstractions $\rightsquigarrow$ **interval** equations more appropriate.
- Non-monotonic equations? Set/**interval** approach. (CPS/CRN, virtual counters, ...)

To solve them, we want some **bound separation** technique.

$$\Phi_{\mathcal{I}} \in \big((\mathbb{Z}^k \to \mathcal{I}(\mathbb{R})) \to (\mathbb{Z}^k \to \mathcal{I}(\mathbb{R}))\big) \;\boldsymbol{\rightarrow}\; (\Phi_{lb}, \Phi_{ub}) \in \big((\mathbb{Z}^k \to \mathbb{R}) \to (\mathbb{Z}^k \to \mathbb{R})\big)^2$$

Interval Recurrence Equations @ Roskilde

To solve them, we want some **bound separation** technique.

$$\Phi_{\mathcal{I}} \in \left((\mathbb{Z}^k \to \mathcal{I}(\mathbb{R})) \to (\mathbb{Z}^k \to \mathcal{I}(\mathbb{R}))\right) \ \boldsymbol{\rightarrow}\ (\Phi_{lb}, \Phi_{ub}) \in \left((\mathbb{Z}^k \to \mathbb{R}) \to (\mathbb{Z}^k \to \mathbb{R})\right)^2$$

- Typical approach is conservative (and not yet well-formalised), especially in presence of some non-monotonicity.
- John has discovered a phenomenon where it is possible to preserve more precision.

  $$\Phi_{\mathcal{I}} \ \boldsymbol{\rightarrow}\ (\Phi_{\downarrow}, \Phi_{\uparrow})$$

  However, its soundness is not yet proven, and seems to rely on the structure of $\Phi_{\mathcal{I}}$.
  Monotonicity properties. Synchronised non-determinism?
- We will investigate the principles underlying this phenomenon: find out correct assumptions, and conclude with a proof.

Thank you for hosting me!

Questions?

(Now or later this month)